Rational Consumers, Learning and Choice Manipulation

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BASIC SIGNAL HEARDING MODEL

Banerjee's (1992)
restaurant herding model with sequential moves and publicly observable signals:

- Unique Decision Variable
- Sharp Changes in Decision Probabilities
- Almost Immediate Decision Herds

Too restrictive?

OUR APPROACH

Generalization of Banerjee's Model

- **Two** Decision **Variables**
- **Signals** received on the **probability** density function of the **second variable**
- **Bayesian Learning** based on received signals
- **Decision** based on **signals** and **observed** first **variables**
- **Smooth** Decision **Probabilities**
- **Different Risk Attitudes**

❖ **Immediate Decision Herds**

MAIN ASSUMPTIONS
GOODS, UTILITIES AND PROBABILITIES

GOODS AND ADDITIVE UTILITIES

$$\forall \langle x_1, \ldots, x_n \rangle \in \prod_{i \leq n} X_i, \;\; u(\langle x_1, \ldots, x_n \rangle) = u_1(x_1) + \cdots + u_n(x_n).$$

PROBABILITY DENSITY FUNCTIONS

For every $i \leq n$, $\mu_i : X_i \to [0,1]$ is either a continuous probability density or a non-degenerated discrete probability density on X_i.

CERTAINTY EQUIVALENT VALUE

For every $i \leq n$, let E_i denote the expected value of u_i

$$E_i = \int_{X_i} \mu_i(x_i) u_i(x_i) dx_i.$$

For every $i \leq n$, ce_i is the element of X_i whose utility $u_i(ce_i)$ equals the expected value of u_i, that is:

$$ce_i = u_i^{-1}(E_i).$$

EXPECTED SEARCH UTILITIES

THE F(X_1) FUNCTION

$F(x_1)$ describes the decision maker's expected utility derived from checking the second characteristic x_2 of good A after observing that the value of the first characteristic is given by x_1.

$$F(x_1) \stackrel{def}{=} \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2))dx_2 + \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2)dx_2$$

where

$$P^+(x_1) = \{x_2 \in X_2 \cap supp(\mu_2) : u_2(x_2) > E_1 + E_2 - u_1(x_1)\}$$

$$P^-(x_1) = \{x_2 \in X_2 \cap supp(\mu_2) : u_2(x_2) \leq E_1 + E_2 - u_1(x_1)\}$$

$P^+(x_1)$ and $P^-(x_1)$ define the set of values for the second x_2 characteristic from good A such that their combination with the observed first x_1 characteristic delivers a respectively higher or lower-equal utility than a randomly chosen good from $\mathcal{G}$.

THE H(X_1) FUNCTION

$H(x_1)$ describes the decision maker's expected utility obtained from checking the first characteristic y_1 of good B after having observed the value of the first characteristic x_1 from good A.

$$H(x_1) \stackrel{def}{=} \int_{Q^+(x_1)} \mu_1(y_1)(u_1(y_1) + E_2)dy_1 + \int_{Q^-(x_1)} \mu_1(y_1)(\max\{u_1(x_1), E_1\} + E_2)dy_1.$$

where

$$Q^+(x_1) = \{y_1 \in X_1 \cap supp(\mu_1) : u_1(y_1) > \max\{u_1(x_1), E_1\}\}$$

$$Q^-(x_1) = \{y_1 \in X_1 \cap supp(\mu_1) : u_1(y_1) \leq \max\{u_1(x_1), E_1\}\}.$$

$Q^+(x_1)$ and $Q^-(x_1)$, define the set of values for the first y_1 characteristic from good B such that they deliver a respectively higher or lower-equal utility than the maximum between the observed first x_1 characteristic from good A and a randomly chosen good from $\mathcal{G}$.

ADDING SIGNALS AND LEARNING

SIGNALS ON THE UNIFORMLY DISTRIBUTED SECOND CHARACTERISTIC

$$f(x_2|\alpha,\beta) = \begin{cases} \frac{1}{\beta-\alpha} & \text{if } x_2 \in [\alpha,\beta], \\ 0 & \text{otherwise.} \end{cases}$$

DOUBLE THE PROBABILITY MASS OF A SIGNAL ON THE UPPER HALF OF THE DENSITY FUNCTION

$$\pi(\theta|x_2) = \begin{cases} \frac{2}{\beta-\alpha} & \text{if } x_2 \in [\frac{\beta}{2},\beta], \\ \frac{1}{2(\beta-\alpha)} & \text{if } x_2 \in [\alpha,\frac{\beta}{2}]. \end{cases}$$

BAYESIAN BELIEFS UPDATE AFTER 1 SIGNAL

$$f(x_2|\theta=1) = \frac{\pi(\theta|x_2)f(x_2)}{\int_{X_2}\pi(\theta|x_2)f(x_2)dx_2}$$

BAYESIAN BELIEFS UPDATE AFTER 2 SIGNALS

$$f(x_2|\theta=2) = \frac{\pi(\theta|x_2)f(x_2|\theta=1)}{\int_{X_2}\pi(\theta|x_2)f(x_2|\theta=1)dx_2}$$

ABSENCE OF LEARNING

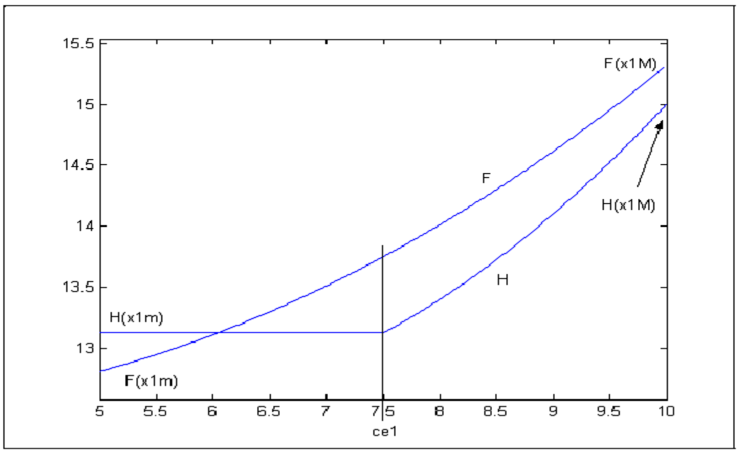
RISK NEUTRALITY

Characteristic spaces: $X_1 = [5,10]$, $X_2 = [0,10]$

Utility functions: $u_1(x_1) = x_1$; $u_2(x_2) = x_2$

Probability densities: both continuous and uniform

$\forall x_1 \in X_1,\ \mu_1(x_1) = \frac{1}{5}$; $\quad \forall x_2 \in X_2,\ \mu_2(x_2) = \frac{1}{10}$



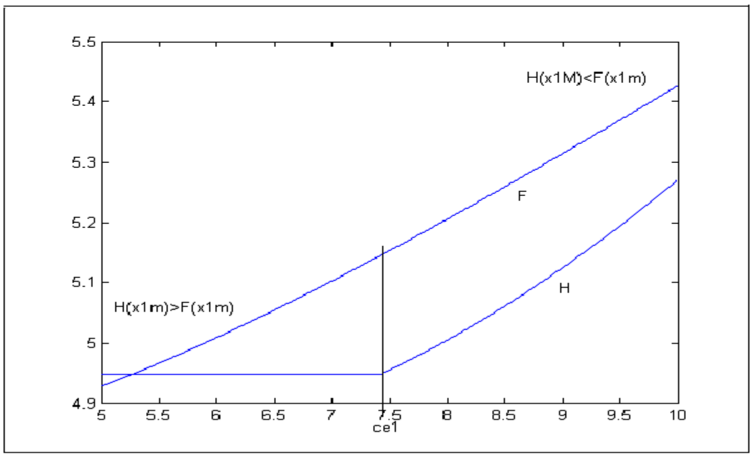
RISK AVERSION

Characteristic spaces: $X_1 = [5,10]$, $X_2 = [0,10]$

Utility functions: $u_1(x_1) = \sqrt{x_1}$; $u_2(x_2) = \sqrt{x_2}$

Probability densities: both continuous and uniform

$\forall x_1 \in X_1,\ \mu_1(x_1) = \frac{1}{5};$ $\forall x_2 \in X_2,\ \mu_2(x_2) = \frac{1}{10}$



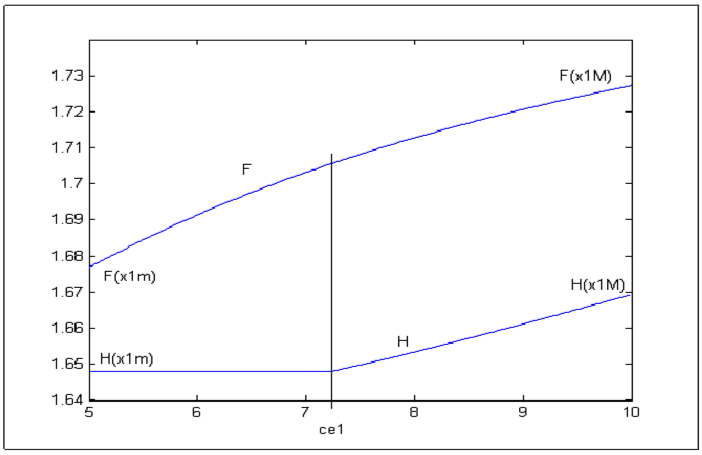
A NATURAL HERD?

Characteristic spaces: $X_1 = [5,10]$, $X_2 = [0,10]$

Utility functions: $u_1(x_1) = \frac{x_1}{x_1+1}$; $u_2(x_2) = \frac{x_2}{x_2+1}$

Probability densities: both continuous and uniform

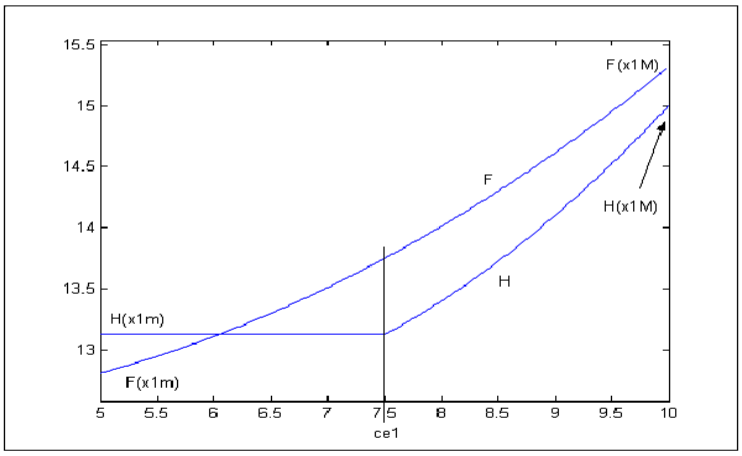
$\forall x_1 \in X_1,\ \mu_1(x_1) = \frac{1}{5};$ $\forall x_2 \in X_2,\ \mu_2(x_2) = \frac{1}{10}$



ENTER LEARNING

NO SIGNAL AND RISK NEUTRALITY

Characteristic spaces: $X_1 = [5,10]$, $X_2 = [0,10]$
Utility functions: $u_1(x_1) = x_1$; $u_2(x_2) = x_2$
Probability densities: both continuous and uniform
$\forall x_1 \in X_1,\ \mu_1(x_1) = \frac{1}{5}$; $\quad \forall x_2 \in X_2,\ \mu_2(x_2) = \frac{1}{10}$



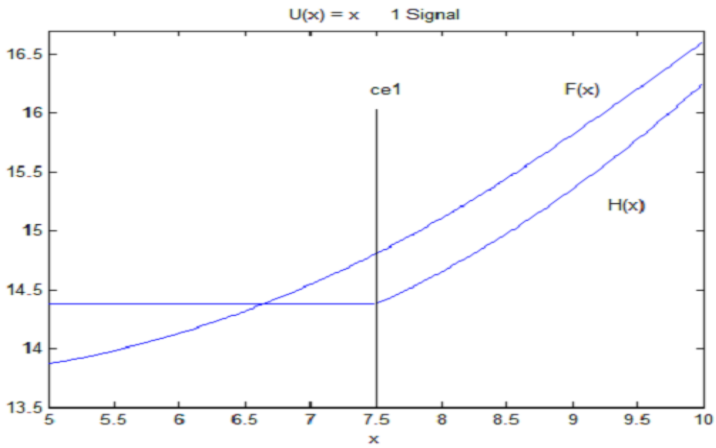
ONE POSITIVE SIGNAL AND RISK NEUTRALITY

Characteristic spaces: $X_1 = [5,10]$, $X_2 = [0,10]$

Utility functions: $u_1(x_1) = x_1$; $u_2(x_2) = x_2$

Probability densities: both continuous and uniform

$\forall x_1 \in X_1$, $\mu_1(x_1) = \frac{1}{5}$; $\forall x_2 \in X_2$, $\mu_2(x_2) = \frac{1}{10}$



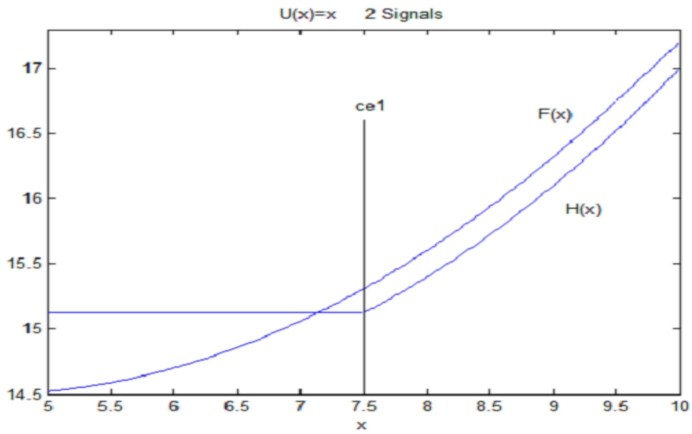
TWO POSITIVE SIGNALS AND RISK NEUTRALITY

Characteristic spaces: $X_1 = [5,10]$, $X_2 = [0,10]$

Utility functions: $u_1(x_1) = x_1$; $u_2(x_2) = x_2$

Probability densities: both continuous and uniform

$\forall x_1 \in X_1$, $\mu_1(x_1) = \frac{1}{5}$; $\forall x_2 \in X_2$, $\mu_2(x_2) = \frac{1}{10}$



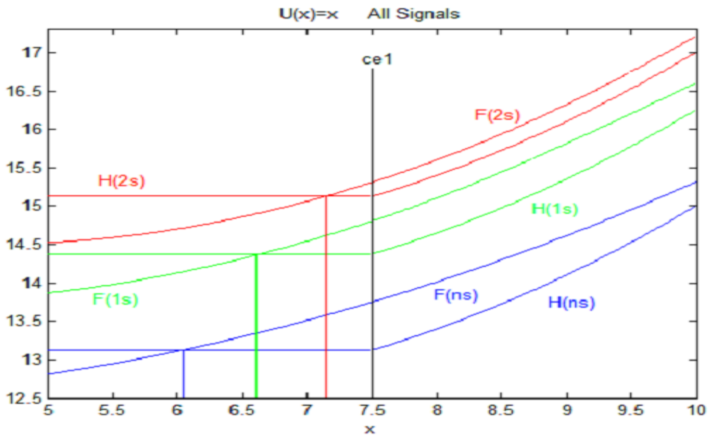
COMPARING SIGNAL EFFECTS

Characteristic spaces: $X_1 = [5,10]$, $X_2 = [0,10]$

Utility functions: $u_1(x_1) = x_1$; $u_2(x_2) = x_2$

Probability densities: both continuous and uniform

$\forall x_1 \in X_1,\ \mu_1(x_1) = \frac{1}{5}$; $\forall x_2 \in X_2,\ \mu_2(x_2) = \frac{1}{10}$



CONCLUSIONS

- A **principal** can easily **issue signals** so as to **manipulate** the **choice** of uninformed but **perfectly rational agents.**

- A **positive relationship** is obtained between the **number of signals issued by a principal** and the **induced bias** on the **choice probability of agents.**

- **Agents** seem to become **less risk averse** as the **number of signals** on a given good **increases.**

- A **set of single signals** issued by a **subset of principals suffices** to generate the **herding mechanism** described by Banerjee within the **entire set of agents.**